Target Positioning Using TOA Measurements

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Introduction

Radio Positioning can be divided into
1. Active Target: target transmits signals
2. Passive Target: target reflects signals
Applications of Passive Radio Positioning
• Localization of Survivors in Emergency Rescue
• Positioning of Intruders
Previous researches on passive positioning:
2. Time of arrival (TOA): No methods can approach CRLB.

Problem & Hypothesis

For small error, we can omit the second order items.

Since there are \( m \) equations like (4), they can be formulated in a matrix form as follows
\[
\mathbf{h} - \mathbf{S} = \mathbf{b}_e,
\]
where
\[
\mathbf{h} = \begin{bmatrix} a_1 h_1 - \hat{c}_1 \\ a_2 h_2 - \hat{c}_2 \\ \vdots \\ a_m h_m - \hat{c}_m \end{bmatrix},
\]
\[
\mathbf{S} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_{mn} \end{bmatrix},
\]
\[
\mathbf{b}_e = \begin{bmatrix} \hat{c}_1 \\ \hat{c}_2 \\ \vdots \\ \hat{c}_m \end{bmatrix}.
\]

Known Information
1. Locations of transmitter and receivers.
2. Signal travel ranges from the transmitter to the receivers.

Objective
- \( \Theta \) to be the target location.
- \( \hat{\Theta} \) to be the estimated location.

Estimation Criteria
- Minimum Square Error

Two Step Expectation Maximization (TSEM) Algorithm – Step 1

The measured signal travel ranges at the receivers are
\[
e = \sqrt{(x_a-x)^2 + (y_a-y)^2} = \text{Distance between Transmitter and Receiver}.
\]

With some manipulations, (1) can be rewritten as
\[
2a_x x_a + 2b_y y_a - 2r = \sqrt{(x-a)^2 + (y-b)^2} - r = \text{Distance between Targets}.
\]

Assume the range measurements are Gaussian distributed
\[
\mathbb{E}[e] = 0, \quad \text{var}(e) = \sigma_e^2.
\]

Substitute (3) into (2), we obtain
\[
\frac{a_x^2 + b_y^2}{2} + a_x x + b_y y - \sqrt{(x-a)^2 + (y-b)^2} = \mathbb{E}[e] = 0.
\]

Two Step Expectation Maximization (TSEM) Algorithm – Step 2

Construct a vector \( \mathbf{g} \) as follows
\[
\mathbf{g} = \Theta - \mathbf{G} \hat{\Theta}
\]
where
\[
\hat{\Theta} = (\hat{x}, \hat{y})^T, \quad \Theta = (x, y)^T
\]
and \( \mathbf{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \)

The optimum \( \hat{\Theta} \) is the one minimizing \( (\Theta - \mathbf{G} \hat{\Theta})^T \Omega (\Theta - \mathbf{G} \hat{\Theta}) \), denoted by
\[
\hat{\Theta} = (\Theta - \mathbf{G} \hat{\Theta})^T \Omega (\Theta - \mathbf{G} \hat{\Theta}).
\]

Finally, the estimate of target location \( (x, y) \) is obtained by
\[
\hat{x} = (x, y) = (x, y, \hat{\Theta}^T, \hat{\Theta}).
\]

The final result is the one of the four options in (13) minimizing the square error as follows
\[
x = \sum_{i=1}^{m} \sqrt{(x^2 + y^2) + \hat{c}_i (x^2 + y^2)}.
\]