Nodal pricing is a widely adopted transmission pricing scheme. Nodal price equals locational marginal price. Real power flow from i to j: \( q_{ij} = -q_{ji} \). Cost-benefit function: \( c_i(q_i) \) (increasing and convex on \( \mathbb{R} \)). Approximate model: AC flow \( \rightarrow \) DC flow.

**Motivation**

- Nodal pricing is a widely adopted transmission pricing scheme.
- Nodal price = locational marginal price.
- Underlying assumption: competitive economic environment.

**From General Equilibrium to Nash**

- Real world: oligopoly or monopoly may exist.
- Generators/consumers have no incentive to reveal their private information (marginal cost/benefit) truthfully.
- Convert the original model to a game.
- Bid space: piecewise linear function.
- Transformation of the dispatch and pricing scheme.
- Focus: Nash equilibrium.
- Main result: nodal pricing is subject to manipulation.
- Counterexample.
  - Assumption: line (2,3) is binding in the efficient dispatch.
  - Claim: Nash equilibrium does not even exist!

**VCG-Type Mechanism**

- Standard VCG mechanism applies.
- However, the type space cannot be parameterized.
- We seek a Nash-implementation mechanism.
- Two-dimensional bid \( b_i = (\beta_i, d_i) \):
  - \( \beta_i \): bid price
  - \( d_i \): maximum quantity
- Generalized mechanism.
  - \( \tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_N) \) as a solution of the following
  
  \[
  \begin{align*}
  & \text{maximize} \sum_{i} \beta_i x_i \\
  & \text{subject to} \quad g_k(x, y) \leq 0, \quad k = 1, \ldots, m \\
  & \quad h_l(x, y) = 0, \quad l = 1, \ldots, p \\
  & \quad x_i \leq d_i, \quad i = 1, \ldots, n \\
  \end{align*}
  \]

  - \( \tilde{x}^{-i} \) as a solution with \( d_i = 0 \) (when \( i \) is not present).
  - Payment made by player \( i \) (externality imposed on others)
    
    \[ w_i = \sum_{j \neq i} \beta_j \tilde{x}_j^{-i} - \sum_{j \neq i} \beta_j \tilde{x}_j \]

  - Player \( i \)'s payoff
    
    \[ u_i = v_i(\tilde{x}_i) - w_i \]

- (Theorem) There exists an efficient Nash equilibrium in the proposed mechanism.
- Apply the generalized mechanism to the power network.
- VCG pricing works, while nodal pricing does not.