A Competitive Rate Allocation Game

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Introduction

- We introduce a competitive rate allocation game in which two receivers compete to forward the data from a transmitter to a destination in exchange for a payment proportional to the amount of forwarded data.
- At each time slot the channel from the transmitter to each receiver is an independent random variable with two states, high or low, affecting the amount of data that can be transmitted.
- Receivers make "bids" on the state of their channel and the transmitter allocates rate accordingly.
- Receivers are rewarded for successful transmissions and penalized for unsuccessful transmissions.

Goal

The goal of the transmitter is to set the penalties in such a way that even if the receivers are selfish, the data forwarded is close to the optimal transmission.

Model

- Channels are independent of each other and the channel states come from an i.i.d distribution.
- \( p_i \): the probability that channel \( i \) is in state high at any time.

Receivers’ perspective

Expected reward table from receivers’ point of view

<table>
<thead>
<tr>
<th>Receiver 2</th>
<th>L</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>((R_1, R_1))</td>
<td>((0, p_1R_2))</td>
</tr>
<tr>
<td>H</td>
<td>((p_1R_3, 0))</td>
<td>((p_1R_2, p_2R_2))</td>
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Transmitter’s perspective

Expected reward table from transmitter’ point of view

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Nash equilibrium

Exist 2 Nash equilibrium

\( b_i = b_j \)

Unique Nash equilibrium

\( b_i = b_j \)

Theorem, Lemma, Corollary

Theorem: If \( c = \frac{R_2 - R_1}{R_2 - R_3} \) and \( D = \frac{R_1}{R_2 - R_3} \), then \( PoA \leq 2 \).

Lemma: For any fixed \( C \) and \( D \), there exists and \( P \) such that \( PoA \) is at least \( 2R_1/R_3 \).

Corollary: \( PoA \) for the rate allocation game over all instances can be arbitrarily close to 2 for any \( C \) and \( D \).

Unknown Channels

**Algorithm**: Online learning using UCB1

\[
\bar{x}_i + D + \frac{2\ln(n)}{n_t} + \frac{2\ln(n)}{n_t}
\]

Simulations

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