Adaptive Vibrational Control for High Accuracy Systems
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Introduction/Applications
Industrial and defense applications of unknown/time-varying periodic disturbance attenuation:
• Laser beam pointing (jitter suppression)
• Structural vibration control
• Noise control of turboprop aircraft
• Vibration reduction in helicopters
• Noise reduction in HVAC systems
• Track following in disk drives
• Suppression of disturbances caused by control moment gyroscopes in spacecraft

Main Results
The proposed adaptive control law guarantees that if the unmodeled dynamics term satisfies some norm-bound condition, then all signals in the closed-loop system are uniformly bounded and the plant output satisfies
\[ \frac{1}{T} \sum_{i=k}^{k+T-1} \|y(i)\|^2 \leq c \|Q(z)\Delta_m(z)G_0(z)\|_{\text{L}_2}\text{H} + cv_0^2 + \frac{c}{T} \]
for any $T; k$ and some finite constant $c$ independent of $T; k$, where $v_0 = \sup|v(k)|$. In addition, in the absence of noise and modeling error, the adaptive law ensures the convergence of $y(k)$ to zero.

Problem Formulation

\[
\begin{align*}
\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} & \rightarrow \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \rightarrow \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \\
d_1 & = \sum a_i \sin(\omega_i k + \varphi_i) + v(k) \\
y_1(k) & = G(z)[u(k)] + d(k) \\
y_2(k) & = (1 + \Delta_m(z))G_0(z)[u(k)] + d(k)
\end{align*}
\]

Objective: To minimize the effect of $d$ on $y$.

Robust Adaptive Control Law
\[
\begin{align*}
\zeta(k) & = y(k) - G_0(z)[u(k)] \\
P^{-1}(k) & = P^{-1}(k-1) + \Phi(k)\Phi^T(k) \\
\varphi^T(k) & = \hat{\theta}^T(k) - \hat{\theta}^T(k-1) + \Phi(k) \\
n_d(k) & = \delta_0 n_d(k-1) + \|\zeta(k)\|^2 \\
\hat{\theta}(k) & = \text{proj}(\hat{\theta}(k-1) + P(k)\Phi(k)\varphi(k)) \\
u(k) & = -\begin{bmatrix} \hat{\theta}^T(k-1)w_1(k-1) + \hat{\theta}^T_2(k-1)w_2(k-1) \\ \hat{\theta}^T_3(k-1)w_1(k-1) + \hat{\theta}^T_4(k-1)w_2(k-1) \end{bmatrix}
\end{align*}
\]

Simulation Results

Output Signal

Summary & Future Work
A robust adaptive control law for suppression of unknown time-varying periodic disturbances for multi-input multi-output linear systems in the presence of unmodeled dynamics has been proposed.

Open Problems:
Unknown time-varying periodic disturbances for:
• Linear time invariant plants with large parametric uncertainties
• Linear time-varying plants
• Uncertain nonlinear plants

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