Statistical Estimation with noisy Rician/non-central chi distributed MR images
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Motivation & Introduction

<table>
<thead>
<tr>
<th>Rician Distributed</th>
<th>Sum of squares (SoS) Acquisition</th>
<th>Non-central Chi Distributed</th>
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</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Histogram Of Magnitude Image" /></td>
<td><img src="image2.png" alt="Magnitude Image" /> + <img src="image3.png" alt="Histogram Of SoS Image" /></td>
<td></td>
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</tbody>
</table>

- Magnitude or SoS data used extensively in quantitative experiments.
- Rician or non-central chi (NCC) Negative log likelihood (NLL) is complicated and non-convex!!!
- Estimation using Gaussian approximation is not accurate in low SNR

New estimation technique:
- Optimizes Rician/NCC NLL
- Iterative method - based on majorize-minimize approach
- Each iteration has a least square formulation

Example Application: Estimation of spherical harmonic coefficients in diffusion MRI

Simulation
- A noisy voxel with two tensors was simulated
- SH coefficients were estimated from this noisy data using the Rician NLL based MM approach.
- Diffusion profile and quantitative parameters like generalized anisotropy and absolute error were calculated.
- Rician NLL based estimation gives more accurate results!!

Quantitative Analysis with multiple two tensor models
- SNR 20
- SNR 8
- SNR 3

GFA
- Angle b/w the tensors

References

Conclusion
- A novel optimization method was developed for estimating Rician/NCC data from its non-convex likelihood.
- Two example applications – denoising and estimation of SH coefficients were demonstrated.

Future work
- Estimate the SH coefficients from multi-shell diffusion data acquisition.

Method

\[ x_{i+1} = \arg \min_{Ax \geq 0} \frac{1}{2\sigma^2} \|Ax - \hat{y}(x_i)\|^2 + R(x) \]

where \( A \) is a dictionary: \( x \rightarrow y, \hat{y} = y^{\frac{2}{2+1}} \), \( y \): Observed image, \( x_{i+1} \): Estimated image at the \((i + 1)\)th iteration, \( R(x) \): regularization

Example Application: Denoising

Simulation

Convergence study

Real Data

5 times Avg Ref Image

Noise Image

MM Tikhonov Regularization

MM TV Regularization

Diffusion Profile (90 degrees)

SNR 6

SNR 2

SNR 1

SNR 3
