Identifiability Results for Ill-posed Bilinear Inverse Problems

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Main Message
- Identifiability crucial in inverse problems
- Not well understood for non-linear systems/constraints
- We develop theory for Bilinear Inverse Problems
- Subsumes blind estimation
- Deterministic characterization of identifiability
- Probabilistic scaling law
- General conic constraints included, e.g., sparsity and low rank constraints
- Connect blind estimation to low-rank matrix recovery
- Readily available convex relaxations

Introduction

Matrix Factorization
Given \( Z = XY \)
Find \( (X, Y) \)
Subject to \( X_{ij}, Y_{jk} \geq 0 \)

Bilinear Map

\( S(x, y) \)
Linear Convolution: \( S(x, y) = x * y \)
\((m = 3, n = 4, p = m + n - 1 = 6)\)

Conic Constraint

\( K \)

\( x^T, S_k \)

\( S \)

Simulation Results

minimize \( \text{rank}(X) \)
subject to \( \|X - M\|_F \leq \epsilon \)
\( \mathcal{L}(X) = 0 \)
- Used Reweighted Nuclear Norm Heuristic
- Used Convolution Operator

Universal Identifiability

\( M \) is domain of ambiguity
\( M' = \{ Y - Z \mid Y, Z \in \mathcal{K}' \} \)
\( \mathcal{N}(\mathcal{L}, 2) \) is rank-2 null space

Instance Identifiability

\( M \) is identifiable
\( M_{id} \) is not identifiable
\( \mathcal{L}(\cdot) \) is null space
\( \mathcal{C}(\cdot) \) is column space

Exponential Scaling Law
- i.i.d. Gaussian/Bernoulli Inputs
- Probability of Identifiability = 
  \( 1 - \exp[C_1 \cdot p - C_2 \cdot (m + n)] \)
- \( p \) is DoF in rank-2 null space
- \( m, n \) are problem dimensions
- \( p = o(m + n) \) implies identifiability w.h.p.

References
- S. Choudhary and U. Mitra, On Identifiability in Bilinear Inverse Problems, ICASSP 2013

Sponsors: ONR N00014-09-1-0700, NSF CNS-0832186 and NSF CCF-1117896
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