Computing encounter distributions of multiple random walkers
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Introduction

• **Goal**: characterize the distribution of encounter times of walkers on random graphs including: exact computation of pairwise inter-encounter time distribution for a particular pair of random walkers, and approximate computation of individual-to-any inter-encounter time (i.e., the time between contacts of a particular walker with any of the other walkers in the population).

• **Motivation**: exploit potential of opportunistic networks (inter encounter time)

Problem Formulation And Modeling

• N walkers walking on the connected graph characterized by V vertexes and S edges.

• For every timeslot, all walkers move on the graph following transition probability matrix \( P \).

\[
A = \{1, 2, 3, \ldots, |V|\} \quad B = \{(1, 1), (1, 2), (1, 3), \ldots, (|V|, |V|)\}
\]

• Interested concepts:
  • PET (Inter Pairwise Encounter Time)
  • IAET (Inter Any Encounter Time)

• \( P(x, y, t) \) is the probability that the walker 1 initially stays at vertex \( x \), walker 2 initially stays at vertex \( y \), they can first meet after \( t \) time steps:

\[
P(x, y, 0) = 1, \forall x \in A
P(x, y, 0) = 0, \forall x, y \in A, x \neq y
P(x, y, t) = \sum_{x', y', t-1 \in B} P(x', y', t-1) \cdot M(x', y')(x, y)
\]

• Inter-pairwise encounter time probability:

\[
P_{PET}(t) = \sum_{z \in A} P(z, z, t) \cdot \pi_z
\]

• The probability given that the walker 1 initially stays at vertex \( x \), walker 2 initially stays at vertex \( y \), the pair hasn’t met for \( t \) time slots:

\[
\overline{P}(x, y, t) = \overline{P}(x, y, t-1) - P(x, y, t), x \in A, y \in A
\]

• Considering all N walkers:

\[
\overline{P}(L_z, t) = \prod_{i=2}^{N} \overline{P}(z, l_i, t)
\]

• The probability that the particular walker meet one of remaining walkers at location \( z \) and hasn’t met any other walkers since then up to time slot \( t \):

\[
\overline{P}(z, t) = \frac{\sum_{i=2}^{N} \overline{P}(L_z, t)}{|V| N - 2}
\]

• Inter any encounter time probability:

\[
P_{IAET}(t) = \sum_{z \in A} P(z, t) \cdot \pi_z
P_{IAET}(t) = \overline{P}_{IAET}(t-1) - \overline{P}_{IAET}(t)
\]

Illustrative example

Simulation results

Extension to multi-communities case

Conclusions

Recursive polynomial-time computation yielding of Pairwise inter-encounter time and approximate computation of inter-any encounter time.

This work was funded in part by NSF via award CNS-1217260