Continuous measurements

Quantum dynamics are reversible, deterministic and continuous. However, quantum measurements are irreversible, non-deterministic, and discontinuous. Can we describe both dynamics and measurement continuously?

Continuous decompositions as random walks

In [OB05] it was shown that any quantum measurement \{M_1, M_2\} can be decomposed into a 1-dimensional continuous stochastic process

\[ M(\psi) = M_1 \]

\[ M(0) = I \]

\[ M(\psi) = M_2 \]

In [FB14] we’ve shown that any qubit-probe interacting in a fixed way with the system being measured could only decompose measurements of the form

\[ M_1 = U_1 (\alpha I_1 + \beta \Pi S) V \]

\[ M_2 = U_2 \left( \sqrt{1 - \alpha^2} I_1 + \sqrt{1 - \beta^2} \Pi S \right) V \]

Linear Hamiltonian control terms

New question: What can be done with a qubit-probe and a tunable interaction?

In the above circuit we define

Probe state: \[ \sigma(x) = |0\rangle \]

Detector states: \[ \sigma^p = |\pm\rangle \]

Tunable interaction: \[ H_{PS} = Y_p \otimes \hat{\epsilon}(x) \]

Linear control terms: \[ \hat{\epsilon}(x) = \sum_{i=0}^{d} p_i(x) H_i \]

Algebra of the control set: \[ F = \text{span} \{ H_1, \ldots, H_d \} \]

Step operators: \[ M_\delta(x) = \frac{1}{\sqrt{2}} \left( I + \delta \hat{\epsilon}(\Pi S) |0\rangle \langle 0| \right) \]

Total walk operator: \[ M(x) \propto \lim_{k \to \infty} \left( I + \delta \hat{\epsilon}(\Pi S) M_\delta \right) \]

Endpoint operators: \[ M_{1,2} \propto \lim_{x \to -x_1} M(x) \]

The above must satisfy the following equations

Reversibility equation: \[ M_\epsilon(x \pm \delta) M_\epsilon(x) \propto I \]

Operator propagation: \[ \partial_x M(x) = -\hat{\epsilon}(x) M(x) \]

Quadratic systems of ODEs

The reversibility equation and the operator propagation equation can be rewritten as quadratic systems of ODEs

Quadratic ODE sys. (1)

\[ \sum_{k=0}^{d} \partial_k p_i(x) H_k = \frac{1}{2} \sum_{j=0}^{d} p_i(x) p_j(x) \{ H_i, H_j \} \]

Quadratic ODE sys. (2)

\[ \sum_{k=0}^{d} \partial_k a_i(x) H_k = -\frac{1}{2} \sum_{j=0}^{d} p_i(x) a_j(x) H \]

Closure lemma

In order to satisfy the reversibility equation, the span of active linear control terms \( F = \text{span} \{ H_i \} \) must be closed under anti-commutation.

Restriction algorithm

Input: \( F = \{U, ZI, ZZ, XX, YY\} \)

Function \( \text{RESTRICT}(F) \)

if \( F^2 = F \) then return \( F \)

else

\( S \leftarrow \text{BASIS} (F^2) \)

for all \( s \in S \) do

\( s \leftarrow \text{RESTRICT}(F^2) \)

end for

return \( S \)

end if

end function

Extension algorithm

Input: \( F = \{U, ZI, ZZ, XX, YY\} \)

Function \( \text{EXTEND}(F) \)

while \( F^2 \supset F \) do

\( F = \text{BASIS} (F^2) \)

end while

end function

Discussion

- Using the control set \( F = \text{span} \{ I, X, Z \} \) we can only decompose measurements of the form achieved by qubit-probe feedback above.
- Quadratic ODE system (2) is completely determined by system (1) and the initial condition \( M(0) = I \).
- Quadratic ODE system (1) contains no orbits [KS95].
- If the span of controls \( F \) is also closed under \( H_1 H_2 H_3 H_4 + H_1 H_2 H_1 \leq F \)
then by the Cohn Reversible Theorem \( F \) is the Hermitian part of the Free algebra generated by \( F \) (i.e.: the most general algebra). [McC04]

References